Grain Size Distribution Documentation of Calculations

The calculations used in the program are fairly simple and for the most part follow ASTM D422. The following sections present the equations used.

1 Moisture Content

Moisture content is calculated with the following formula:

$$MC = 100\% * \frac{W_{wt} - W_{dt}}{W_{dt} - W_t} \tag{1.1}$$

Where:

MC = the moisture content
W_{wt} = the weight of the moisture specimen with tare
W_{dt} = the weight of the dried specimen with tare
W_t = the weight of the container

2 Sieve Test Calculations

2.1 Wash Test

$$PF_{200} = \frac{W_{ts} - W_{wash}}{W_{ts}} * 100\% \tag{1.2}$$

Where:

PF_{200} = the percent of material finer than the #200 sieve
W_{ts} = the total sample weight
W_{wash} = the after-wash weight

The program charts PF_{200} as the #200 percentage if either no further sieve test data are entered (i.e., only a #200 wash test is performed), or the sieve test does not include a #200 sieve.

2.2 Cumulative Weight Retained Method

$$PF = \left(1 - \frac{W_c - W_{ct}}{W_{ts}}\right) * 100\% \tag{1.3}$$

Where:

PF = the percent finer
W_c = the cumulative weight retained
W_{ct} = the tare weight of the cumulative pan
W_{ts} = the total sample weight

An example calculation:

Cumulative weight retained = **1915.2** grams
Cumulative pan tare = **382.5** grams
Sample weight = **1671.4** grams

$$PF = \left(1 - \frac{1915.2 - 382.5}{1671.4}\right) * 100\% = 8.3\%$$

2.3 Per-Sieve Weight Retained Method

$$PF = 1 - \frac{(W_{st} - W_t) + W_{ls}}{W_{ts}} * 100\% \tag{1.4}$$

Where:

W_{st} = the weight of a sieve and its retained material
W_t = the weight of the sieve
W_{ls} = the total amount of material retained on all larger sieves
W_{ts} = the weight of the total sample

An example calculation:

To calculate the percent finer for the third largest sieve in a sieve nest, we need:
Total sample weight = **11.94** grams
Weight retained + tare for the third largest sieve = **9.66** grams
Tare weight for the third largest sieve = **4.19** grams
Material retained on the larger sieves: **0.00** grams on the largest + **0.54** grams on the second largest.

$$PF = 1 - \frac{(9.66 - 4.19) + 0.54}{11.94} * 100\% = 49.7\%$$

2.4 Sample Splits

If the sample is split, the subsequent percent finer values are found as follows:

$$PF_{tot} = \frac{CBT - WR}{DWT} \tag{1.5}$$

Where:

PF_{tot} = the overall percent finer
WR = the weight retained of the split sample

DWT, the post-split sample dry weight, is calculated as follows:

$$DWT = \frac{SGDW}{PF_{ss} - PFW} \tag{1.6}$$

Where:

SGDW = the split gradation dry sample weight
PF_{ss} = the overall percent finer than the split sieve
PFW = the percent washed out from the #200 wash test (or 0 if a wash test was not performed)

CBT, the biased total weight, is calculated as follows:

$$CBT = SGDW + PFW * DWT \tag{1.7}$$

An example calculation:

PF_{ss} = **72.3** %
SGDW = **1871.30** grams
PFW = **0.00** %
WR = **422.00** grams

$$DWT = \frac{1871.30}{72.30 - 0.00} = 25.90$$

$$CBT = 1871.30 + (0.00 * 25.9) = 1871.30$$

$$PF = \frac{1871.30 - 442.00}{25.90} = 55.2\%$$

3 Hydrometer Test Calculations

3.1 Particle Size

$$PS = \sqrt{\frac{30 * v * l}{980 * (GS - GW) * ET}} \tag{1.8}$$

Where:

Ps = the particle size in mm.
v = the fluid viscosity in centipoises
L = the effective depth in cm.
GS = the specific gravity of the soil particles
GW = the specific gravity of water, corrected for temperature
Et = the elapsed time in minutes

For 152H hydrometers, L, the effective depth, is calculated as follows:

$$L = 16.295 - 0.164 * Rm \tag{1.9}$$

Where:

Rm = the hydrometer reading corrected by subtracting the absolute value of the meniscus correction

This equation is equivalent to the 152H equation given by ASTM D422 et al.:

$$L = L_1 + 0.5 * (L_2 - \frac{V_b}{A}) \tag{1.10}$$

Where:

$$L_1 = \frac{50 - Rm}{50 - 0}(10.5 - 2.3) + 2.3 \tag{1.11}$$

$L_2 = 14.0cm.$
$V_b = 67.0cm.^3$
$A = 27.8cm.^2$

Substituting, D422's effective depth equation becomes:

$$L = [\frac{50}{50} * (10.5 - 2.3) - Rm/50 * (10.5 - 2.3) + 2.3] + [0.5 * 14 - \frac{0.5 * 67.0}{27.8}] \tag{1.12}$$

which, after simplifying, is identical to our equation.

For 151H hydrometers L is calculated with this equation (with D422 derivation similar to the prior 152H equation):

$$L = 16.295 - 0.2645 * Rm \tag{1.13}$$

⇒ Note that the effective depth equation can be changed by the user on a per-test basis.

v, the fluid viscosity, is calculated as:

$$v = C1 + T * (C2 + T * (C3 + T * (C4 + T * C5))) \tag{1.14}$$

Where:

T = the fluid temperature, in degrees Celsius
C1 = 0.01732483379693
C2 = -5.041574656095E-04
C3 = 8.387438669317E-06
C4 = -7.401129271698E-08
C5 = 2.625994080072E-10

GW, the specific gravity of water, is calculated as:

$$GW = C1 + T * (C2 + T * (C3 + T * C4)) \tag{1.15}$$

Where:

T = the fluid temperature, in degrees Celsius
C1 = 0.99991003252
C2 = 0.00005201921
C3 = -0.00000751229
C4 = 0.00000003605183

An example calculation:

ET = **`8`** minutes
Temp = **`23.5`** Celsius
R = **`34`**
Cm = **`1`**
GS = **`2.7`**
Rm = **`35`** (=R + Cm)
GW = **`0.997452`** (calculation not shown)
v = **`0.00925`** (calculation not shown)
L = **`10.56`** (calculation not shown)

$$PS = \sqrt{\frac{30 * 0.00925 * 10.56}{980 * (2.7 - 0.9975) * 8}} = 0.0148 \text{ mm.}$$

3.2 Percent Finer

For 152H hydrometers, the percent finer than a given opening size is calculated as:

$$PF = \frac{Rc * a}{WB} * 100\% \tag{1.16}$$

Where:

PF = the percent finer
Rc = the corrected hydrometer reading
a = the specific gravity of solids correction factor
WB = the biased hydrometer sample weight

For 151H hydrometers, the calculation is:

$$PF = \frac{100 * GS}{WB * (GS - 1)} * Rc \tag{1.17}$$

Where:

PF = the percent finer
Rc = the corrected hydrometer reading
WB = the biased hydrometer sample weight
GS = the soil specific gravity

When using automatic temperature correction, the corrected hydrometer reading (Rc) is calculated as follows:

$$Rc = R + Ct + Cc \tag{1.18}$$

Where:

R = the actual hydrometer reading (in thousandths for 151H)
Cc = the composite correction at 20 degrees Celsius, as entered by the user
Ct = the composite correction

For 152H hydrometers, Ct, the composite correction, is calculated as follows:

$$Ct = -12.35952257 + T * (1.51062059 + T * (-0.06923056 + T * 0.00122483)) \tag{1.19}$$

Where:

T = the fluid temperature, in degrees Celsius

For 151H hydrometers, Ct is calculated as:

$$Ct = -7.6338851 + T * (0.93361976 + T * (-0.04284159 + T * 0.000758977)) \tag{1.20}$$

Where:

T = the fluid temperature, in degrees Celsius

When using the multi-point (linear) temperature correction, the corrected hydrometer reading is calculated as follows:

$$Rc = R + Ct \tag{1.21}$$

Where:

R = the actual hydrometer reading (in thousandths for 151H)
Ct = the temperature correction, as interpolated from a linear regression line constructed from the pairs of temperature and reading values entered by the user into the hydrometer correction grid.

- For test temperatures less than the lowest temperature entered into the correction grid, the program will use the correction value corresponding to the lowest correction temperature entered; likewise, for test temperatures higher than the highest temperature entered into the correction grid, the program will use the correction value corresponding to the highest correction temperature entered.

The specific gravity correction factor is:

$$a = \frac{0.6226415 * GS}{GS - 1} \tag{1.22}$$

Where:

GS = the specific gravity of the solids

The biased sample weight is calculated as:

$$WB = \frac{Whs * 10000}{Pss * (100 + Mh)} \tag{1.23}$$

Where:

WB = the biased sample weight, in grams
Whs = the air dry hydrometer sample weight
PSS = the percent passing the separation sieve
Mh = the hygroscopic moisture content per ASTM D422 § 8

An example calculation using a 152H hydrometer:

ET = `8` minutes
Temp. = `23.5` degrees Celsius
R = `34`
GS = `2.7`
Mh = `3.5%`
Linear correction pairs: (-6.0 at 20°), (-5.6 at 22°), (-4.7 at 25°)

By interpolation, Cc at 23.5° = -5.15

$$Rc = 34 + (-5.15) = 28.9$$

$$a = \frac{0.6226415 * 2.7}{2.7 - 1} = 0.989$$

$$WB = \frac{51.7 * 10000}{100 * (100 + 3.5)} = 50.0 \text{ grams}$$

$$PF = \frac{28.9 * 0.989}{50.0} = 57.0\%$$

An additional calculation, using a 151H hydrometer:

ET = **15** minutes
Temp = **22** Celsius
R = **21.5**
GS = **2.65**
WB = **63.5** grams
Cc at 22 degrees Celsius = **-2.2**

$$Rc = 21.5 + (-2.2) = 19.3$$

$$PF = \frac{100 * 2.65}{63.5 * 1.65} * 19.3 = 48.8\%$$

3.3 Calculation of Fractional Components

The fractional components and percentage diameters (D_{85}, D_{60},D_{50}, etc.) are computed by creating a cubic spline model of the particle size distribution curve then solving the model for the curve values at various percentages.

The classification coefficients C_c and C_u are calculated as follows:

$$C_c = \frac{D_{30} * D_{30}}{D_{60} * D_{10}} \tag{1.24}$$

$$C_u = \frac{D_{60}}{D_{10}} \tag{1.25}$$

3.4 Tables of Constants

Table 1.1: Correction Factor for Specific Gravities Other than 2.65 when using Hydrometer 152H

SPECIFIC GRAVITY	CORR. FACTOR	SPECIFIC GRAVITY	CORR. FACTOR
2.50	1.038	2.68	0.994
2.51	1.036	2.69	0.992
2.52	1.033	2.70	0.989
2.53	1.030	2.71	0.987
2.54	1.028	2.72	0.985
2.55	1.025	2.73	0.983
2.56	1.022	2.74	0.981
2.57	1.020	2.75	0.979
2.58	1.017	2.76	0.977
2.59	1.015	2.77	0.975
2.60	1.012	2.78	0.973
2.61	1.010	2.79	0.971
2.62	1.008	2.80	0.969
2.63	1.005	2.81	0.967
2.64	1.003	2.82	0.965
2.65	1.001	2.83	0.963
2.66	0.998	2.84	0.962
2.67	0.996	2.85	0.960

Table 1.2: Automatic Temperature Correction Factor as a Function of Temperature

TEMP, DEG C.	SPECIFIC GRAVITY OF WATER	VISCOSITY OF WATER	TEMP. CORR. FACTOR FOR 152H	TEMP. CORR. FACTOR FOR 151H
15.0	0.99912	0.01141	-1.14	-0.71
15.5	0.99050	0.01126	-1.02	-0.63
16.0	0.99897	0.01111	-0.90	-0.55
16.5	0.99889	0.01097	-0.78	-0.48
17.0	0.99880	0.01083	-0.67	-0.41
17.5	0.99871	0.01069	-0.56	-0.35
18.0	0.99862	0.01056	-0.46	-0.28
18.5	0.99853	0.01043	-0.35	-0.22
19.0	0.99843	0.01030	-0.25	-0.16
19.5	0.99834	0.01017	-0.15	-0.09
20.0	0.99823	0.01005	-0.04	-0.03
20.5	0.99813	0.00993	0.07	0.04
21.0	0.99802	0.00981	0.18	0.11
21.5	0.99791	0.00969	0.29	0.18
22.0	0.99780	0.00958	0.41	0.25
22.5	0.99769	0.00947	0.53	0.33
23.0	0.99757	0.00936	0.66	0.41
23.5	0.99745	0.00925	0.80	0.50
24.0	0.99733	0.00914	0.95	0.59
24.5	0.99721	0.00904	1.11	0.69
25.0	0.99708	0.00894	1.27	0.79
25.5	0.99695	0.00884	1.45	0.90
26.0	0.99682	0.00874	1.64	1.02
26.5	0.99668	0.00864	1.85	1.15
27.0	0.99655	0.00855	2.07	1.28
27.5	0.99641	0.00846	2.30	1.43
28.0	0.99627	0.00836	2.55	1.58
28.5	0.99613	0.00827	2.81	1.75
29.0	0.99598	0.00818	3.10	1.92
29.5	0.99583	0.00809	3.40	2.11
30.0	0.99568	0.00801	3.72	2.31

Table 1.3: Effective Depth for 152H and 151H Hydrometers

Rm	EFFECTIVE DEPTH	Rm	EFFECTIVE DEPTH	Rm	EFFECTIVE DEPTH	Rm	EFFECTIVE DEPTH
0	16.3	26	12.0	0	16.3	20	11.0
1	16.1	27	11.9	1	16.0	21	10.7
2	16.0	28	11.7	2	15.8	22	10.5
3	15.8	29	11.5	3	15.5	23	10.2
4	15.6	30	11.4	4	15.2	24	9.9
5	15.5	31	11.2	5	15.0	25	9.7
6	15.3	32	11.0	6	14.7	26	9.4
7	15.1	33	10.9	7	14.4	27	9.2
8	15.0	34	10.7	8	14.2	28	8.9
9	14.8	35	10.6	9	13.9	29	8.6
10	14.7	36	10.4	10	13.7	30	8.4
11	14.5	37	10.2	11	13.4	31	8.1
12	14.3	38	10.1	12	13.1	32	7.8
13	14.2	39	9.9	13	12.9	33	7.6
14	14.0	40	9.7	14	12.6	34	7.3
15	13.8	41	9.6	15	12.3	35	7.0
16	13.7	42	9.4	16	12.1	36	6.8
17	13.5	43	9.2	17	11.8	37	6.5
18	13.3	44	9.1	18	11.5	38	6.2
19	13.2	45	8.9	19	11.3		
20	13.0	46	8.8				
21	12.9	47	8.6				
22	12.7	48	8.4				
23	12.5	49	8.3				
24	12.4	50	8.1				
25	12.2						